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Application of Assignment Problem and Traffic Intensity in Minimization of Traffic Congestion

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ABSTRACT

In our daily life, we are facing several problems; Traffic congestion is one of them, which is becoming serious day by day. High number of vehicles on road, insufficient infrastructure, unreasonable developments are some reasons for increasing traffic congestion. The aim of this paper is to introduce a mathematical model which can be applied to traffic control system having fixed time signal with preset time to minimize traffic congestion at an intersection. In this paper, we use the concept of traffic intensity and Assignment model for traffic congestion minimization.

Keywords: Traffic intensity, Assignment model, Traffic congestion

A.M.S. subject classification (2000)-90B22 ,60K25,60K30.

INTRODUCTION

An efficient traffic system plays a very important role in smooth running of road activities. Traffic congestion is mostly common in highly populated areas with small land mass. Some reasons of traffic congestion are:

1. Traffic accident
2. Road construction or maintenance.
3. Wrong driving habits.
4. Insufficient driver knowledge.
5. Violation of traffic rules.
6. Increase in no. of vehicles.
7. Insufficient infrastructure.
8. Bad condition of roads.
9. No separate lanes for heavy vehicles like trucks, loading vehicles etc.
10. Fluctuation (sudden increase in traffic) due to holidays.

Problems occur due to traffic congestion:

1. It adversely affects the economic productivity.
2. Increase fuel consumption.
3. Increase road user’s frustration which may sometime lead to driver being involved in road rage & cause damage to vehicles, injuries to driver & other people.
4. Waste of time as well as money.
5. Affects human health, environment and economy.
6. Affects essential or emergency services.

For avoiding such a type of critical conditions, necessary & effective measures are needed.

Few of them are:

1. Separate lanes with increased road capacity can be constructed for commercial vehicles.
2. Carpooling and use of public transport (if possible) will help in traffic congestion reduction.
3. Optimization of traffic lights system.
4. Strict implementation of traffic rules can also have a great impact on the traffic amount on the road.
5. The roads can be configured with movable barrier that helps in changing the vehicle lanes based on traffic demand.
6. Adaptive signal technology can be used for reduction of traffic congestion.
7. C
onstruct flyovers for those areas which are prone to traffic congestion.
8. Service lane's & bye-pass can be constructed to those roads/lanes which have high- traffic intensity.

Mala at **el** [11] discuss the contribution and application of queuing theory in the field of traffic congestion. Ankush Kumar at **el** [9] put light on Traffic congestion and its possible solution in the urban transportation system. Martin Ankoye et **al** [3] discuss the application of queuing theory in minimization of traffic caused by vehicles on roads of Kumasi metropolis of Ghana as a case study. Dirk Heidemann [5] derived some new analytical results on statistical distribution of queue length and delays at traffic. Nico Vandaele et **al** [12] put a review on queuing approach to traffic flows in great detail.

In queuing theory, A/B/C/D/E/F (known as Kendall's notation) is the standard form to classify and describe any queuing model[1]. The first three characters were introduced by D.G .Kendall in 1953[6].Later, A. Lee. in 1966 [2] added fourth, fifth and sixth character.

THEORETICAL MODEL

In our model, vehicles are considered as customer & Traffic light is considered as a server. Here the work of server is to regulate the flow of vehicles through the roads/lane's connected to an intersection. Arrival of vehicles follows random fashion, and they are made to line up according to their order of arrival i.e., they adopt FCFS model. Arrival of customers follows Poisson's distribution, and they form a single channel. Service of vehicles is done independently in a parallel manner according to the order of their arrival. The inter- arrival time between two successive arrivals and between two successive services of vehicles is considered as independent and is distributed normally

[4]. Since, the arrival of vehicles on the road is not limited, therefore, we can consider it as infinite calling.

So, we can apply here (M/M/1) :(∞ /FCFS) model; It is a Probabilistic queuing model.

Here: -

1. First M denotes exponential distribution of arrival time or Poisson's distribution of arrivals [1].
2. Second M denotes exponential service time distribution. Here letter M is used to represent Markovian property of the exponential process [1].
3. 1 represent single server/service station [1].
4. ∞: Infinite calling Population [1].
5. FCFS: The service discipline is first come first serve.

Let λ=average rate of arrival of vehicles on the road, μ=average service rate of vehicles.

Then, Traffic intensity=$\rho=\frac{\lambda}{\mu}$,where $\frac{\lambda}{\mu}<1$.

The traffic intensity of any lane can be easily calculated by finding the arrival and services of vehicles per unit time on the road through observation (daily/random).

We know that, for a system to be in a steady state the value of ρ must be less than 1 because, if ρ> 1, the arrival rate will be greater than the rate of service and then the queue length will be infinite, and the system will not remain in the steady state. Here we are using the same concept i.e. For a stable traffic flow system, $\rho=\frac{\lambda}{\mu}<1$, means the service rate μ should always be greater than the arrival rate λ & hence ρ<1.On this basis, we can easily divide the stability of the traffic flow system in following categories.

1. If ρ<5, then it represents that system is highly stable & traffic flow is very smooth.
2. If 5 ≤ρ≤6, then it shows the system is stable & traffic flow is smooth.
3. If 6<ρ≤7, then it shows the system is relatively stable & traffic flow is unsmooth.

As the value of ρ goes on increasing, the traffic flow will become critical & as it reaches near to unity then it becomes a worst condition of traffic. One can easily calculate the traffic intensity for any lane by daily or random observation of arrival & service rate of vehicles on that lane/road. "Rowland J.O. Ekeocha & Victor I. Ihebom[4] discuss the use of queuing theory & traffic intensity on some areas of Victor Island ,Lagos.

In this paper, we will discuss the use of Assignment model along with the concept of traffic intensity in minimization of traffic congestion. Traffic signals play a very important role for a stable traffic flow. They are generally classified into 3 categories:

1. Traffic control system: It's working is based on 3-color light system in which Red light is used to indicate "stop". Yellow light indicates that the transition should slow down while green light means vehicles are" allowed to run". Some signals are fixed time signals having preset time i.e., light is operated on a preset time system. However, it has some drawbacks; traffic light will not adjust according to the traffic situation because of its fixed timing which may cause traffic congestion while, on the other hand, when there is very low traffic on the road then a long red light will unnecessarily waste road users time & tests their patience. Second are traffic response signals, which contains expensive equipment's like sensor & detectors that automatically adjust time according to traffic condition. Third type is manually operated signals in which traffic condition is adjusted by traffic police.
2. Pedestrian signals: This type of signals is developed for pedestrian guidance.
3. Flashing signals: It gives warning to vehicle users that if the red signal flashes it means to stop your vehicle before stop line & flashing of yellow signal means run vehicles with caution.

ASSIGNMENT PROBLEM

Assignment problem is a special type of linear programming problem in which is the objective is to find the optimum allocation of a given number of tasks (jobs) to an equal no. of facilities (person).

Effectiveness matrix

Jobs

		1	2	3	………………	j	……………..	n
	1	C_{11}	C_{12}	C_{13}	……………	C_{1j}	……………..	C_{1n}
	2	C_{21}	C_{22}	C_{23}	………………	C_{2j}	……………..	C_{2n}
	3	C_{31}	C_{32}	C_{33}	………………	C_{3j}	……………..	C_{3n}
	.	.	.	.	…………………	.	……………..	.
Persons	.	.	.	.	..……………….	.	……………..	.
	.	.	.	.	……………….	.	……………..	.
	i	C_{i1}	C_{i2}	C_{i3}	……………	C_{ij}	……………..	C_{in}
	.	.	.	.	……………….	.	……………..	.
	.	.	.	.	..……………….	.	……………..	.
	n	C_{n1}	C_{n2}	C_{n3}	..…………	C_{nj}	……………..	C_{nn}

Mathematical formulation of an Assignment problem:

Mathematically an assignment problem can be stated as follows:

Minimize the total assignment cost,

$Z=\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}.x_{ij}$

Where

$x_{ij}=$

$$\begin{cases} 1 \ ,if \ i^{th} \ person \ is \ assigned \ to \ the \ j^{th} \ job \\ 0 \ ,if \ i^{th} \ person \ is \ not \ assigned \ to \ the \ j^{th} \ job \end{cases}$$

Subject to the constraints

1. $\sum_{j=1}^{n} x_{ij}$=1, which represents that only one job is done by the i^{th} person, i=1,2,3……..n.
2. $\sum_{i=1}^{n} x_{ij}$=1, which represents that only one person should be assigned to the j^{th} job, j=1,2,3,……,n.

An assignment problem can easily be solved by Hungarian method. This method is actually an algorithm which is developed by Harold Kuhn [7][8]. He named the algorithm as "Hungarian method", because a large part of this algorithm is based on earlier work of two Hungarian mathematicians: Denes Konig and Jeno Egervary. Larry J. Leblanc at **el** [10] discuss a solution technique for large-scale road network equilibrium assignment and related flow problems with nonlinear costs.

METHODOLOGY

Let us consider there are three lanes L_1,L_2,L_3 connected to an intersection each having traffic control system associated with Red, Yellow, Green light with it. Now find out the preset time of Red, Yellow, Green light associated with each lane, which can easily be determined by simply observation. The Preset time of traffic light system for each lane may be same. Now, construct an assignment/effectiveness matrix by taking the three colors lights on one side and available lanes on another side of the matrix. Taking C_{ij} as the preset time assigned to each light for a particular lane, where i=1,2,3……..n & j=1,2,3………n. Now, interchange the value of C_{ij} in the matrix so that maximum possible combinations of matrices will form. Now solve each assignment matrix individually by Hungarian method & find the minimum congestion time for each matrix. The matrix which has minimum traffic congestion time will select as required assignment model & apply it in traffic control system having fixed time signal with preset time. If there is a tie between the matrices for minimum traffic congestion time, then any, one of the two matrices can be selected randomly.

If the number of lanes associated with an intersection is greater than the number of the lights, then add a dummy light to make assignment matrix balanced and vice-versa and after that solve it. However, this method is tedious and time taking. So, if we use the concept of traffic intensity in it, then it makes our work easier i.e. if we know the traffic intensity of each lane associated with an intersection, then we will select that lane/lanes which have high- traffic intensity i.e. which have high traffic congestion problem, & after that interchange the values of C_{ij} for only those lanes. This method will reduce the number of obtained assignment matrix & then solve it by Hungarian method. Now, select that matrix which has minimum traffic congestion time as our required model. We can try to understand it by an example:

1. Let three lanes L_1,L_2,L_3 are connected to an intersection. Each lane is associated with a separate but same traffic control system containing Red, Yellow, Green lights. It has been observed that lanes L_2 & L_3 have stable & smooth traffic flow, but lane L_1 has unstable & critical traffic flow i.e. this lane has high traffic intensity. Find how the assignment should be done to minimize total traffic congestion time at the given intersection, where

	Traffic Light		
Lane	Red	Yellow	Green
L_1	3	2	1
L_2	3	2	1
L_3	3	2	1

Solution: Here, C_{ij} ,i=1,2,3 & j=1,2,3 represents the preset time for each light in a single phase. It is clear that that all the 3 lanes are associated with traffic control system having same preset time for each phase. Now if we apply our 1st method and interchange all the values of C_{ij} given in the matrix then we get a large number of combinations of possible outcome matrices. The solution of such a large number of matrices by Hungarian method is a very tedious job. But it has been given that out of the 3 lanes L_1 has high traffic intensity i.e., L_1 has heavy traffic congestion problem. So, we will interchange only values of C_{11} , C_{12} , C_{13} & then we get total 6 possible assignment matrices which we can easily solve.

(1)

Lane	Red	Yellow	Green
L_1	3	2	1
L_2	3	2	1
L_3	3	2	1

(2)

Lane	Red	Yellow	Green
L_1	3	1	2
L_2	3	2	1
L_3	3	2	1

(3)

Lane	Red	Yellow	Green
L_1	1	3	2
L_2	3	2	1
L_3	3	2	1

Lane	Red	Yellow	Green

(4)

Lane	Red	Yellow	Green
L_1	1	2	3
L_2	3	2	1
L_3	3	2	1

(5)

Lane	Red	Yellow	Green
L_1	2	1	3
L_2	3	2	1
L_3	3	2	1

(6)

L_1	2	3	1
L_2	3	2	1
L_3	3	2	1

On solving each of the above assignment matrices by Hungarian method, we find that matrix 3 and matrix 4 have same minimum congestion time equals to 4 minutes. So, required assignment models are:

Lane	Traffic light (Time Required)
L_1	Red (1 minute)
L_2	Yellow(2minutes)
L_3	Green (1 minute)

or

Lane	Traffic light (Time Required)
L_1	Red(1 minute)
L_2	Yellow (1minute)
L_3	Green(2 minutes)

CONCLUSION

This study concludes how an assignment model can be applied to Traffic control system to minimize traffic congestion system at an intersection. It also reveals that if we already calculate or know the traffic intensity of the existing lanes, then above-mentioned assignment method can more effectively be applied in short time with less effort. This method does not require any expensive equipment's. It can be used to those intersections where lanes are associated with Traffic control system having fixed time signal with preset time. However, challenges may occur when traffic intensity or traffic

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